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Absolute surface metrology by shear rotation with position error correction

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Abstract

Background: Absolute test is one of the most important and efficient techniques to saperate the reference surface which usually limits the accuracy of test results.

Method: For the position error correction in absolute interferometry tests based on rotational and translational shears, the estimation algorithm adopts least-squares technique to eliminate azimuthal errors caused by rotation inaccuracy and the errors of angular orders are compensated with the help of Zernike polynomials fitting by an additional rotation measurement with a suitable selection of rotation angles.

Results: Experimental results show that the corrected results with azimuthal errors are very close to those with no errors, compared to the results before correction.

Conclusions: It can be seen clearly that the testing errors caused by rotation inaccuracy and alignment errors of the measurements can be consequently eliminated from the differences in measurement results by the proposed method.

Keywords: Absolute test, Shear rotation, Error correction, Zernike polynomials

Background

In optical interferometric testing, the test surface map is not obtained independently but only in combination with the reference surface. Several ingenious techniques have been devised to obtain absolute surface measurements, e.g., two-sphere [1, 2] method for spherical reference surfaces and "three-flat" approach for flat surface [3]. However, the classic two-sphere method with cat's-eye position measurement is sensitive to the lateral shear of the coma wavefront, which will introduce astigmatism and spherical terms [2]. For decades, the shift-rotation methods without the testing of cat's-eye position have been developed to test spherical and flat surfaces [4–9]. These approaches yield an estimate for the test surface errors without changing experimental settings, such as cavity length, that may affect the apparent reference errors. The classic multi-angle averaging method proposed by Evans and Kestner, measures the spherical surface at N angular positions equally spaced with respect to the optical axis and the resulting wavefronts are averaged, then errors in the rotated member with angular orders that are not integer multiples of the number of positions will be removed without Zernike fitting [10, 11].

It always assumes that there is no azimuthal position error during part rotation in the previous absolute test methods. However, the rotations of the test part introduce uncertainties related to azimuthal errors of the rotational angle and lateral displacement of the part with respect to the optical axis of the interferometer [11]. Moreover, rotation should be very precise when higher order spatial frequency terms are required, which are particularly sensitive to azimuthal position errors. In practice, there are challenges to rotate the test surface accurately to the desired positions, especially for large optics, and keep the environment and metrology system stable during the multi-measurements [12]. So we present a method to determine the true azimuthal positions of part rotation and consequently eliminate testing errors caused by rotation inaccuracy.

Method

The shearing test is based on the analysis of differences in measurement results that occur when rotating or translating the test surface. The test results

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yield a collection of error maps. Each error map describes the sum of apparent reference errors and test surface errors for a particular position and orientation of the test surface. If the test part is rotated to N equally spaced positions about the optical axis and the resulting, we can get the averaged wavefront

$$T_{ave}(\rho,\theta) = \frac{1}{N}\sum_{i=0}^{N-1} T_i(\rho,\theta) = \frac{1}{N}\sum_{i=0}^{N-1}[R(\rho,\theta)+S(\rho,\theta)] \quad (1)$$

where $R(\rho, \theta)$ is the systematic error including the reference surface, $S(\rho, \theta)$ is the surface error of the test part.

The wavefront of circular cross section can be expanded by polar coordinate polynomials in the following form

$$W(\rho,\theta) = \sum_{k,l} R_l^k(\rho)\left(\alpha_l^k \cos k\theta + \alpha_l^{-k}\sin k\theta\right) \quad (2)$$

where $R_l^k(\rho)$ are the radial terms of Zernike polynomials and coefficients $\alpha_l^{\pm k}$ specify the magnitude of each term while the angular terms specify the angular part of the polynomial representation. ρ and θ are the normalized radial and angular coordinates.

From Eq. (2), if the wavefront is rotated to N equally spaced positions about the optical axis ($\phi = 2\pi/N$), the averaged resulting wavefront can be written as

$$W_{ave}(\rho,\theta) = \frac{1}{N}\sum_{j=0}^{N-1} W\left(\rho, \theta + j\frac{2\pi}{N}\right) = \frac{1}{N}\sum_{j,k,l}^{N-1} R_l^k(\rho)\left(\cos k\theta \sum_{j=0}^{N-1}\alpha_l^{j\phi,k} + \sin k\theta \sum_{j=0}^{N-1}\alpha_l^{j\phi,-k}\right) \quad (3)$$

where

$$\begin{bmatrix}\alpha_l^{\phi,k}\\ \alpha_l^{\phi,-k}\end{bmatrix} = \begin{bmatrix}\cos k\phi & \sin k\phi\\ -\sin k\phi & \cos k\phi\end{bmatrix}\begin{bmatrix}\alpha_l^k\\ \alpha_l^{-k}\end{bmatrix} \quad (4)$$

For $k = 0$ (i.e., for rotationally symmetric terms), it is the intuitively obvious result that the procedure has no influence on rotationally symmetric terms. For $k \neq 0$, the series sum to zero for all $\cos k\phi$ except $k = cN(i = 1,2,3\ldots)$ and for all $\sin k\ \phi$. It is easy to see that rotating a wavefront to N equally spaced positions and averaging removes nonrotationally symmetric terms of all angular orders except $kN\theta$. The term $W_{kN\theta}(\rho, \theta)$ is the Nth rotationally symmetric component (angular orders $kN\theta$), which can be written as

$$W_{kN\theta}(\rho,\theta) = \sum_{k,l}(-1)^{k(N+1)} R_l^{kN}(\rho)(\alpha_l^{kN}\cos kN\theta + \alpha_l^{-kN}\sin kN\theta) \quad (5)$$

So the averaged test wavefront can be rewritten as

$$T_{ave}(\rho,\theta) = R(\rho,\theta) + S_{sym}(\rho,\theta) + W_{kN\theta}(\rho,\theta) \quad (6)$$

where $S_{sym}(\rho, \theta)$ is the rotational symmetry surface deviation of the test part $S(\rho, \theta)$.

Furthermore, the asymmetric component of the test surface can be derived as

$$S_{asy}(\rho,\theta) = T_i(\rho,\theta) - T_{ave}(\rho,\theta) + W_{kN\theta}(\rho,\theta) \quad (7)$$

The errors of angular variation $kN\theta$ can be represented based on Zernike polynomials and additional shear rotation measurement [9]. And it may be always neglected in the multi-angle averaging method, when N is large enough.

Additional measurements provide redundancies to improve and characterize measurement uncertainties. However, the rotation of the test part also introduces uncertainties related to azimuthal errors of the rotational angle and lateral displacement of the part with respect to the optical axis of the interferometer. The effect of uncertainties will arise from uncertainties in the rotational angle. Moreover, there are challenges to rotate the test surface accurately to the desired positions, especially for large optics, and keep the environment and metrology system stable during the multi-measurements.

So the estimation algorithm should be presentd to eliminate azimuthal errors caused by rotation inaccuracy. And the unknown relative alignment of the measurements also can be estimated through the differences in measurement results at overlapping areas.

The difference W between the shear rotation measurements can be written as

$$\begin{aligned} W &= R(\rho,\theta) + S_i(\rho,\theta) - R(\rho,\theta) - S_j(\rho,\theta+\phi) \\ &= S_i(\rho,\theta) - S_j(\rho,\theta+\phi) \\ &= \sum_{k,l} R_l^k(\rho)\left(\Delta\alpha_l^k\cos k\theta + \Delta\alpha_l^{-k}\sin k\theta\right) \end{aligned} \quad (8)$$

where $\Delta\alpha_l^{\pm k}$ is the differences of the coefficients between two measurements.

It is trivially obvious to find $\alpha_l^{\pm k}$ in terms of $\Delta\alpha_l^{\pm k}$ from the difference of two measurements from Eqs. (4) and (8)

$$\alpha_l^{\pm k} = -\frac{1}{2}\left[\Delta\alpha_l^{\pm k} \pm \frac{\Delta\alpha_l^{\mp k}\sin k\phi}{(1-\cos k\phi)}\right] \quad (9)$$

This shows that the azimuthal terms of the wavefront can be determined from the azimuthal terms of the difference between the original wavefront and itself after rotation by ϕ. So the wavefront can be represented based on Zernike polynomials. Futermore, the $kN\theta$ variations of surface deviation $W_{kN\theta}(\rho,\theta)$ neglected in the multi-angle averaging method can also be obtained by additional rotation testing with a suitable selection of rotation angles θ_0 with $k = cN$ and $k\theta_0 \neq 2m\pi$ (m is an integer).

The differences of the coefficients between two measurements can be written as

$$\Delta\alpha_l^{\pm k} = \alpha_l^{\pm k}(\cos k\phi_i - 1) \pm \alpha_l^{\mp k}\sin k\phi_i \tag{10}$$

For azimuthal position error correction, the angle ϕ_i can be treated as additional unknowns together with the coefficients $\alpha_l^{\pm k}$. Then their actual values can be determined from the measured difference wavefront by least-squares method. Then the estimation algorithm adopts least-squares technique to eliminate azimuthal errors caused by rotation inaccuracy.

From Eq. (8), the wavefront difference can be further written as

$$\begin{aligned} W_i^k &= \sum_{k,l} R_l^k(\rho)\{(\cos k\phi_i - 1)\left(\alpha_l^k \cos k\theta + \alpha_l^{-k}\sin k\theta\right) \\ &\quad + \sin k\phi_i\left(\alpha_l^{-k}\cos k\theta + \alpha_l^k \sin k\theta\right)\} \\ &= \sum_{k,l}\{\gamma_{0l}^k Z_l^k(\rho,\theta)(\cos k\phi_i - 1) \\ &\quad + \tilde{\gamma}_{0l}^k Z_l^k(\rho,\theta)\sin k\phi_i\} = \sum_{k,l}\xi_{li}^k Z_l^k(\rho,\theta) \end{aligned} \tag{11}$$

The cost functions can be obtained by least squares method and be minimized to determine the true values of the unknowns of γ_{0l}^k, $\tilde{\gamma}_{0l}^k$ and ϕ_i, as discussed in [12].

$$\begin{bmatrix} \sum_{i=0}^{N-1}[\cos(k\phi_i)-1]^2 & \sum_{i=0}^{N-1}\sin(k\phi_i)[\cos(k\phi_i)-1] \\ \sum_{i=0}^{N-1}\sin(k\phi_i)[\cos(k\phi_i)-1] & \sum_{i=0}^{N-1}\sin^2(k\phi_i) \end{bmatrix} \begin{bmatrix} \gamma_{0l}^k \\ \tilde{\gamma}_{0l}^k \end{bmatrix} = \left[\sum_{i=0}^{N-1}\hat{X}_{li}^k[\cos(k\phi_i)-1] \; \sum_{i=0}^{N-1}\hat{X}_{li}^k\sin(k\phi_i)\right] \tag{12}$$

$$\begin{bmatrix} \sum_l^{L(k)}[\gamma_{0l}^k]^2 & \sum_l^{L(k)}\gamma_{0l}^k\,\tilde{\gamma}_{0l}^k \\ \sum_l^{L(k)}\gamma_{0l}^k\,\tilde{\gamma}_{0l}^k & \sum_l^{L(k)}[\tilde{\gamma}_{0l}^k]^2 \end{bmatrix} \begin{bmatrix} \cos(k\phi_i) \\ \sin(k\phi_i) \end{bmatrix} = \begin{bmatrix} \sum_l^{L(k)}\left\{\hat{X}_{li}^k\gamma_{0l}^k + [\gamma_{0l}^k]^2\right\} \\ \sum_l^{L(k)}\left\{\hat{X}_{li}^k\hat{\gamma}_{0l}^k + \gamma_{0l}^k\hat{\gamma}_{0l}^k\right\} \end{bmatrix} \tag{13}$$

This generalized algorithm adopts least-squares technique to determine the true azimuthal positions of part rotation and consequently eliminates testing errors caused by rotation inaccuracy. The true values of the unknowns of γ_{0l}^k, $\tilde{\gamma}_{0l}^k$ and ϕ_i can be obtained by the iterative procedure. The total computational time is influenced by the number of terms of Zernike polynomials in consideration (maximum l and k), the number of rotation N, and the precision of the initial guess of ϕ_i. Finally, the testing errors caused by rotation inaccuracy can be compensated by the solutions of γ_{0l}^k, $\tilde{\gamma}_{0l}^k$ and ϕ_i.

Results

For the verification of the described method, experiments are presented in a standard Fizeau interferometer. The surface under test is a spherical mirror with a clear aperture of 100 mm and surface error within λ/10PV. The accuracy of rotations can be better than 0.1° and the 5-Axis Mount of ZYGO Company can provide 13 mm X and Y adjustment, 50 mm Z adjustment and ±2° tip and tilt adjustment. The spherical surface is tested at the normal testing position and various orientations with the classic multi-angle averaging method. These approaches can yield an estimate for the test surface errors without changing experimental settings, such as cavity length, that may affect the apparent reference errors.

The averaged wavefronts for $N = 6$ and 12 are shown in Fig. 1. The errors of angular orders $kN\theta$ resembling a hexagon can be seen obviously from Fig. 1a, which may introduce unnecessary measurement errors when it is neglected in the absolute surface metrology. When N is large enough, the terms 2nπ/ϕ are close to rotationally symmetric deviations, as shown in Fig. 1b. The errors of angular orders $kN\theta$ can be quite small.

The differences of Fig. 1a and b are shown in Fig. 2a. It can see clearly that the averaged wavefront are suffering from the errors of angular orders $kN\theta$. For the compensation of $W_{kN\theta}(\rho,\theta)$, the additional rotation testing with a suitable selection of rotation angles is implemented. The $W_{kN\theta}(\rho,\theta)$ of the test surface are restructured and compensated with the help of least-squares fitting of Zernike polynomials. The differences of Fig. 1a and b after $W_{kN\theta}(\rho,\theta)$ compensation can be seen from Fig. 2b, and the compensated errors of angular variation $kN\theta$ are shown in Fig. 3. The differences of Fig. 1a and b after compensation are very small. The errors of angular variation $kN\theta$ have been well compensated. It implies that the described method with $W_{kN\theta}(\rho,\theta)$ compensation can obtain high accuracy even with fewer rotation measurements. However, because of position errors, the errors caused by rotation inaccuracy still can be seen from Fig. 2b.

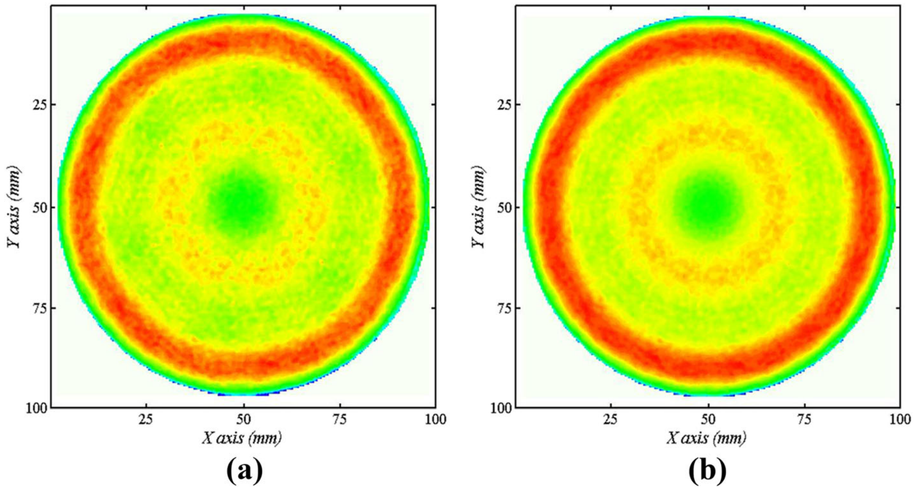

Fig. 1 The averaged wavefront ($N = 6$ and 12). **a** N=6, PV=44.10nm, RMS=5.50nm **b** N=12, PV=37.90nm, RMS=4.75nm

Furthermore, the averaged wavefront for $N = 6$ with position errors (azimuthal errors and alignment error) introduced is shown in Fig. 4 and the difference of the averaged wavefront for $N = 6$ before and after position errors introduced is shown in Fig. 5. Figures 1a and 4 have a similar distribution on optical path difference and some differences on PV and RMS. More details can be seen from Fig. 5. The test results are suffering from the position errors. As mentioned above, it's difficult to rotate the test surface accurately to the desired positions, especially for large optics. There are also many challenges to keep the environment and metrology system stable during the multi-averaging measurements, especially for large N. So the position error correction is necessary.

Discussion

In order to correct the errors due to the rotation inaccuracy, the estimation algorithm adopts least-squares technique to determine the true azimuthal positions of part rotation and consequently eliminates testing errors caused by rotation inaccuracy. The surface is tested on the precision rotation stage with accurate position and random azimuthal errors within ±2° respectively.

The Zernike coefficients of the results in absolute surface metrology are shown in Fig. 6. The Zernike coefficients with

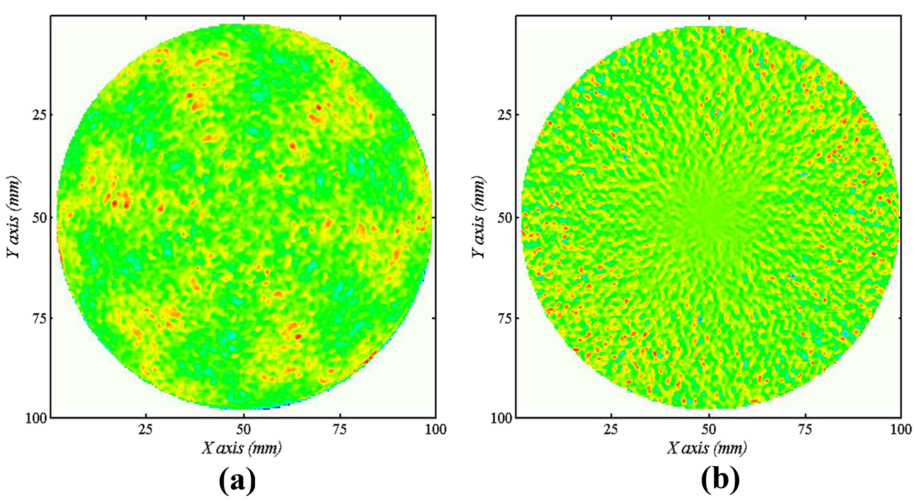

Fig. 2 Differences of the averaged wavefront between $N = 6$ and $N = 12$ before and after compensation. **a** Before compensation, PV=8.27nm, RMS=0.80nm **b** After compensation, PV=6.30nm, RMS=0.70nm

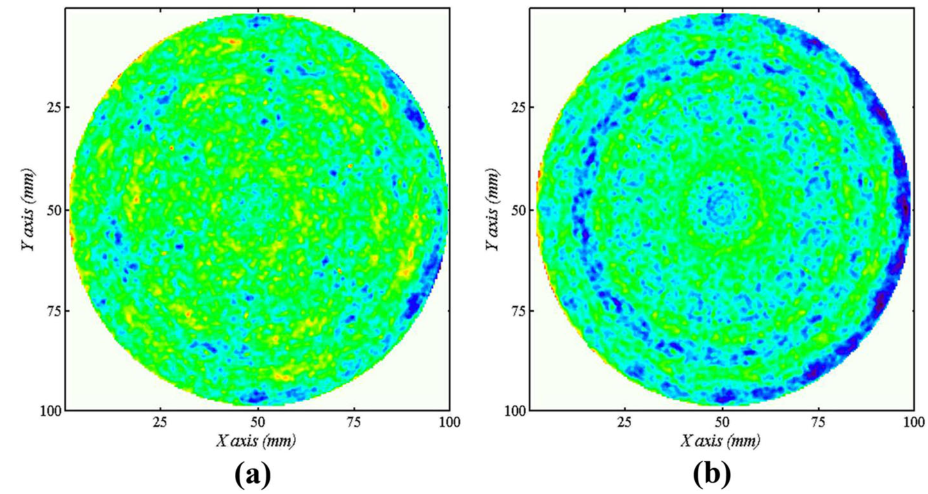

Fig. 3 Compensated errors of angular variation kNθ. **a** N=6, PV=8.22nm, RMS=1.07nm **b** N=12, PV=6.30nm, RMS=0.81nm

the correction of azimuthal errors and alignment errors are also shown in Fig. 6. The corrected Zernike coefficients are very close to those with fine adjustment and no additional azimuthal errors, compared to the results before correction. The coma terms (Z7 ~ Z8, Z14 ~ Z15, Z23 ~ Z24) and spherical terms (Z9, Z16, Z25, Z36) introduced by the azimuthal errors and alignment errors have been well suppressed. It implies that the testing errors caused by rotation inaccuracy and alignment errors of the measurements can be consequently eliminated from the differences in measurement results by the proposed method.

Conclusions

We discussed the position error estimation algorithm to determine the true azimuthal positions of part rotation and the $kN\theta$ compensation method to offer possibility to obtain high accuracy even with fewer rotation

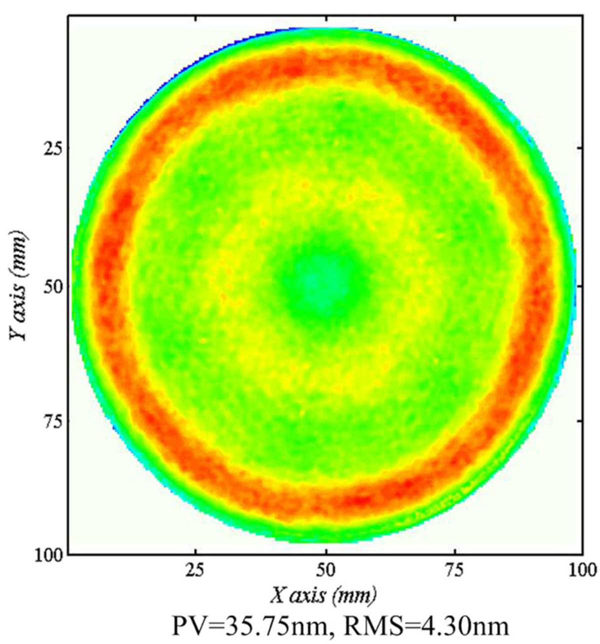

Fig. 4 The averaged wavefront for N = 6 with position errors

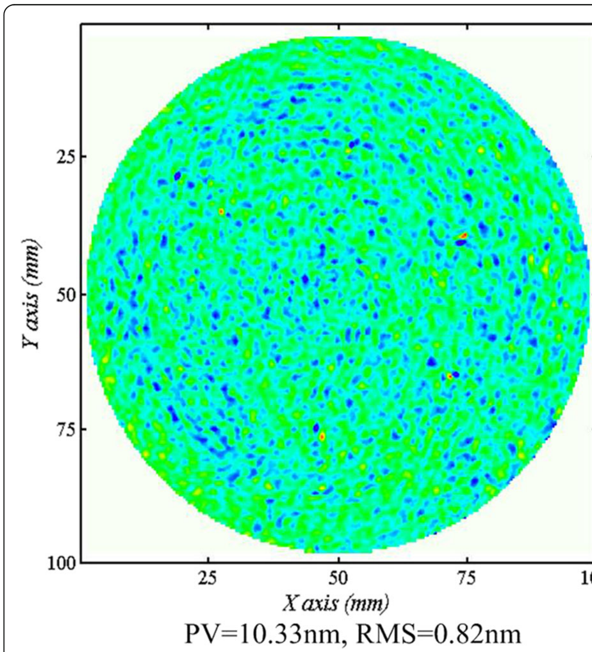

Fig. 5 The difference of the averaged wavefront for N = 6 before and after position errors introduced

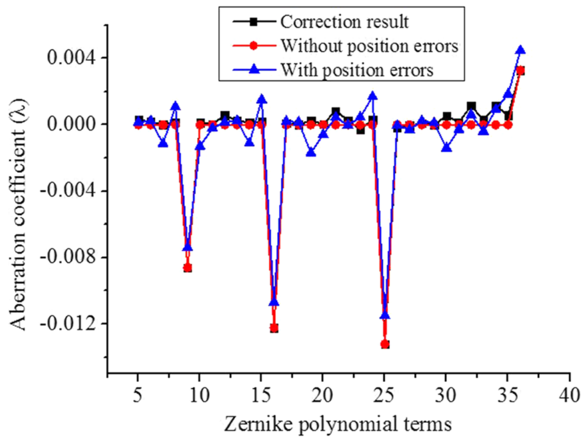

Fig. 6 Coefficients of Zernike polynomials (λ = 632.8 nm)

measurements. It can be used to overcome the challenges of rotating the test surface accurately to the desired positions, especially for large optics and obtain the higher order spatial frequency terms required. Experimental results have been given to verify the effectiveness of the proposed method.

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Authors' contributions
All authors have participated in the method discussion and result analysis. The experiments are conducted by JT. All authors have read and agreed with the contents of the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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